SOLUTIONS MANUAL

to

ROCKET PROPULSION ELEMENTS, 8th EDITION

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This manual is in part an outgrowth of courses taught by Prof. Biblarz, both at the senior/graduate university level and as short courses. All solutions in this manual were prepared by the authors with some student collaboration. A number of design-type- problems are included in several of the book’s chapters and solutions to these were largely prepared by George Sutton. The style in this manual is informal.

The solutions given are mostly complete but not all problems are included, particularly in the more applied chapters. For problems which are of a “design nature” more than one answer is possible and expected. A few other problems, as presently stated, need for additional information or call for assumptions or value-estimates to get started, otherwise the answers can only be given in parametric form. For the most part, all tabular and unit-conversion information may be found in the book but not necessarily in the same chapter as the problem. There are also several problems that require fundamentals not found explicitly in our book – here other standard references on the rocket field should be consulted before searching for specific papers from the research literature.

We have attempted to be clear and accurate in the preparation of this manual. Should you discover errors or deficiencies, or wish to make technical comments about specific solutions, please contact Oscar Biblarz by e-mail at: obiblarz@nps.edu or info@oscarbiblarz.com or by regular mail at: Code ME/Bi, Department of Mechanical & Astronautical Engineering, Naval Postgraduate School, Monterey, CA 93943-5146. Any contributions to problems (particularly those presently without a solution) would be most welcome and could appear in subsequent versions of this manual.

It is our intention that, with rare exceptions, this manual will be made available only to qualified academics. If you have concerns about the distribution of this manual or publisher related items please contact Robert Argentieri, Executive Editor, John Wiley & Sons, at bargenti@wiley.com.
1. A jet of fluid hits a stationary flat plate in the manner shown.
(a) If there is 50 kg of fluid flowing per minute at an absolute velocity of 200 m/sec, what will be the force on the plate?
(b) What will this force be when the plate moves in the direction of flow at $u = 50$ km/h? Explain the methodology.

\[ m(v - u) = F_x \]
\[ O = F_y \]
\[ m = \rho A(v - u) \]

a) $m = 50$ kg/min, $c = 200$ m/sec, $u = 0$
\[ F_x = \frac{50}{60}(200) = 166.7 \text{ N on plate} \]
b) $\rho A = \frac{m}{c} = \frac{50}{60}/200 = 0.042 \text{ kg/m}$
\[ u = 50 \text{ km/h} = 13.9 \text{ m/sec} \]
\[ F_x = 0.042(200 - 13.9)^2 = 144.24 \text{ N on plate} \]

In Part b), the mass flow rate on the plate has decreased because of the decreased relative velocity. The flow is incompressible.

2. The following data are given for a certain rocket unit: thrust, 8896 N; propellant consumption, 3.867 kg/sec; velocity of vehicle, 400 m/sec; energy content of propellant, 6.911 MJ/kg. Assume 100% combustion efficiency.

Determine (a) the effective velocity; (b) the kinetic jet energy rate per unit flow of propellant; (c) the internal efficiency; (d) the propulsive efficiency; (e) the overall efficiency; (f) the specific impulse; (g) the specific propellant consumption.

\[ F = 8896 \text{ N}, \quad m = 3.867 \text{ kg/sec}, \quad u = 400 \text{ m/sec}, \quad Q_R = 6.911 \text{ MJ/kg}, \quad \eta_{comb} = 1.0 \]

a) $c = \frac{Fm}{m} = \frac{8896}{3.867} = 2,300 \text{ m/sec}$

b) $\frac{E_{jet}}{m} = \frac{P_{jet}}{m} = \frac{0.5mv^2}{m} = 0.5c^2 = 2.645 \times 10^6 \text{ m}^2/\text{sec}^2$ or 2.645 MJ/kg

c) $\eta_{int} = \frac{0.5mc^2}{mQ_R\eta_{comb}} = \frac{c^2}{2Q_R} = 0.383$

d) $\eta_p = \frac{2u/c}{1 + (u/c)^2} = \frac{2(400/2300)^2}{1 + (400/2300)^2} = 0.3376$

e) $\eta = \eta_p\eta_{int} = 12.9 \%$

OR $\eta = \frac{Fm}{mQ_R + 0.5mu^2} = 13.3 \%$
f) \( I_s = \frac{F}{mg_0} = \frac{8896}{3.867 \times 9.81} = 234.5 \text{ sec} \)

g) \( SFC = 1/I_s = 0.00426 \text{ sec}^{-1} \)

3. A certain rocket has an effective exhaust velocity of 7000 ft/sec; it consumes 280 lbm/sec of propellant mass, each of which liberates 2400 Btu/lbm. The unit operates for 65 sec. Construct a set of curves plotting the propulsive, internal, and overall efficiencies versus the velocity ratio \( u/c (0 < u/c < 1.0) \). The rated flight velocity equals 5000 ft/sec. Calculate (a) the specific impulse; (b) the total impulse; (c) the mass of propellants required; (d) the volume that the propellants occupy if their average specific gravity is 0.925.

[Assume sea-level operation where 1 lbm weights 1 lbf so there is no numerical distinction between the two units.]

c = 7000 ft/sec, \( \dot{m} = 280 \text{ lbm/sec} \), \( Q_R = 2400 \text{ Btu/lbm} \), \( t_p = 65 \text{ sec} \), \( u_{RATED} = 5000 \text{ ft/sec} \).

a) \( I_s = c/g_0 = 7000/32.2 = 214.4 \text{ sec} \).

b) \( I_t = Ft_p = c \dot{m} t_p/g_0 = (7000)(280/32.2)(65) = 3.96 \times 10^6 \text{ lbf-sec} \)

c) \( w = \dot{w} t_p = (280)(65)(32.2/32.2) = 18,200 \text{ lbf} \) or 18,200 lbm at sea level

d) \( SG = (\text{density})/(\text{density of liquid water at standard conditions}) = 0.925 \)
\[ \rho = (0.925)(62.4) = 57.72 \text{ lbm/ft}^3 \text{ and Volume} = 18,200/57.72 = 315.8 \text{ ft}^3 \]

4. For the rocket in Problem 2, calculate the specific power, assuming a propulsion system dry mass of 80 kg and a duration of 3 min.

\[ \text{DRY MASS} = \text{EMPTY MASS} \text{ (see Fig. 4-1)} \]
Specific Power \( = \frac{P_{jet}}{m_0} = \frac{0.5Fc}{m_f + m_p} \)

\[ m_0 = (3.867)(180) + 80 = 776.06 \text{ kg} \]

\[ \frac{P_{jet}}{m_0} = \frac{0.5(8896)(2300)}{776.06} = 13.18 \text{ kW/kg} \]

5. A Russian rocket engine (RD-110) consists of four nonmoveable thrust chambers supplied by a single turbopump. The exhaust from the turbine of the turbopump then drives four vernier chamber nozzles (which can be rotated to provide some control of the flight path). Using the information below, determine the thrust, effective exhaust velocity, and mass flow rate of the four vernier thrusters.

Individual thrust chambers (vacuum):

\[ F = 73.14 \text{ kN}, \ c = 3279 \text{ m/sec} \]

Overall engine with verniers (vacuum):

\[ F = 297.93 \text{ kN}, \ c = 3197 \text{ m/sec} \]

(a) vernier thrust ; \( F_v = 297.93 - 4 \times 73.14 = 5.37 \text{ kN} \)

(b) vernier mass flow rate:

\[ \dot{m}_v = \frac{F_v}{c_{oa} - F_v/c_c} = \frac{297930/3197 - 4 \times 73140/3279}{3.97} = 3.97 \text{ kg/sec} \]

(c) vernier effective exhaust velocity:

\[ c_v = \frac{F_v}{\dot{m}_v} = 5370/3.97 = 1353 \text{ m/sec} \]

6. A certain rocket engine has a specific impulse of 250 sec. What range of vehicle velocities \( u \), in units of ft/sec would keep the propulsive efficiencies at or greater than 80%. Also, how could rocket–vehicle staging be used to maintain these high propulsive efficiencies for the range of vehicle velocities encountered during launch?

\[ I_s = 250 \text{ sec}, \ c = (32.17)(250) = 8,042.5 \text{ ft/sec} \]

a) By inspection of Fig. 2-4, \( \eta_p \geq 0.8 \) for \( \frac{1}{2} \leq u/c \leq 2 \)

So, \( 4,021 \leq u \leq 16,085 \text{ ft/sec} \)

b) Design upper stages with increasing \( I_s \) to keep \( u/c \leq 2.0 \) within atmospheric flight.

7. Plot the variation of the thrust and specific impulse against altitude, using the atmospheric pressure information given in Appendix 2, and the data for the Minuteman first-stage rocket thrust chamber in Table 12-3. Assume that \( p_2 = 8.66 \text{ psia} \).

NOTE: Minuteman First Stage data give on Table 12-3 is interpreted here for a problem belonging in Chapter 2. The actual missile-motor’s performance data could be different. The desired result is to find the numbers corresponding to Figure 2-2.
For $p_2 = 8.66$ psia, the design altitude ($p_3 = p_2$) corresponds to $h = 194,000$ ft. Otherwise, $p_3$ is either sea level ($h = 0, p_3 = 14.7$ psia) at launch or zero in the vacuum of space (which the first stage might not reach). The thrust at the design altitude could be taken as 194,600 lbf (average) and the nozzle exit area could be taken as 1642 in$^2$. Using Appendix 2 to convert atmospheric pressure to altitude ($h$), the following equation will generate the plot, with $p_3$ in psia,

$$F = F_{opt} - (p_2 - p_3)A_2 = 194,600 - (8.66 - p_3) \times 1642 \text{ (lbf)}$$

The specific impulse requires knowledge of the flow rate or alternatively the design value. This value could be taken as 254 sec, so the equation becomes, using the $F$ calculated above,

$$I_s = (254/194,600) \times F \text{ (sec)}$$

[SEE FIGURE 2-2 FOR GENERAL TRENDS]

8. During the boost phase, the three Space Shuttle main engines (SSMEs) operate together with the two solid propellant rocket motors (SRBs) for the first 2 minutes. For the remaining thrust time, the SSMEs operate alone. Using Table 1-4, calculate the overall specific impulse for the vehicle during the 2-minute combined thrust operation.

The individual mass flow rates for each propulsion type may be inferred from Table 1-4.

Main engines: $m = 3 \times (1670,000 \text{ N})/(455 \text{ sec}) \times (9.81 \text{ m/sec}^2) = 1,122 \text{ kg/sec}$

SRBs: $m = 2 \times 14,700,000/292 \times 9.81 = 9,137 \text{ kg/sec}$

Using Equation 2-26, we find

$$(I_s)_{ss} = \frac{\Sigma F}{g_0 \Sigma m} = \frac{(3\times1670 + 2\times14700)\times10^3}{9.81 \times (1122 + 9137)} = 342 \text{ sec}$$

This is higher than the advertised 310 sec because the $I_s = 455$ sec in Table 1-4 for the main engines somewhat overestimates the first 2 minutes of operation.

9. For the values given in Table 2-1 for the various propulsion systems, calculate the total impulse for a fixed propellant mass of 2000 kg.

Use upper values from Table 2-1, and take $m_p = 2,000 \text{ kg}$.

$$I_t = (m_pg_0)I_s = 19,620 \times I_s \text{ N-sec}$$

<table>
<thead>
<tr>
<th>Type</th>
<th>$I_s$ (sec)</th>
<th>$I_t$ (N-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid</td>
<td>200</td>
<td>3.92 \times 10^6</td>
</tr>
<tr>
<td>Liquid</td>
<td>410</td>
<td>8.04 \times 10^6</td>
</tr>
<tr>
<td>Monopropellant</td>
<td>223</td>
<td>4.38 \times 10^6</td>
</tr>
<tr>
<td>Method</td>
<td>Impulse</td>
<td>Impulse</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>Nuclear Fission</td>
<td>860</td>
<td>$1.17 \times 10^7$</td>
</tr>
<tr>
<td>Resistojet</td>
<td>300</td>
<td>$5.89 \times 10^6$</td>
</tr>
<tr>
<td>Arcjet-thermal</td>
<td>1200</td>
<td>$2.35 \times 10^7$</td>
</tr>
<tr>
<td>EM, incl. PPT</td>
<td>2500</td>
<td>$4.91 \times 10^7$</td>
</tr>
<tr>
<td>Hall Effect</td>
<td>1700</td>
<td>$3.34 \times 10^7$</td>
</tr>
<tr>
<td>Ion</td>
<td>5000</td>
<td>$9.81 \times 10^7$</td>
</tr>
<tr>
<td>Solar heating</td>
<td>700</td>
<td>$1.37 \times 10^7$</td>
</tr>
</tbody>
</table>

The total impulse is related to the change of vehicle velocity as is done in Chapter 4 but in this problem there is an arbitrarily fixed propellant mass which precludes further conclusions.